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A DECOUPLED LINEAR PROGRAMMING TECHNIQUE FOR POWER SYSTEM STATE ESTIMATION

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Abstract - This paper presents a decoupled linear programming approach to solve the power system state estimation problem. The original problem is decoupled into two L.P. subproblems, namely P-θ and Q-V. These are solved in a sequential manner until the final solution is reached. The elements of the Jacobean matrix are approximated to become constants which need to be evaluated once only at the beginning of the first iteration. This leads to a significant reduction in computer storage and solution time. The accuracy of the final state variables is unaffected by the approximations.

INTRODUCTION

The classical weighted least squares (WLS) method has been widely used to solve the power system state estimation problem. The WLS method takes into account all available information and compromises a solution point which satisfies the least squares criterion. Hence, all measurements contribute towards reaching the solution point. Some of these measurements could be 100% in error (lost measurement) or even 200% in error (wrong sign of measurement). The weights of the different measurements are proportional to the squares of their residuals. Hence, the error in any measurement has a squared effect in attracting the solution point away from the true value. Different prefiltering techniques have been suggested for processing the error-contaminated data before presenting it to the state estimator, such that line loadings are checked for compatibility with available information of voltage levels and circuit breaker on/off positions. This is to detect and exclude large measurements errors. Other bad data detection is performed through the inspection of the residuals after estimation. The effects of bad data on the final solution point can also be minimised by means of bad data suppression technique [1]. Kotinga and Vidyasagan [3] claimed that the weighted least absolute value (WLAV) technique possessed better bad data rejection properties. In general extensive experience is needed before such methods can be properly implemented.

A recent, but different approach, using linear programming (L.P.) to tackle the state estimation problem has been developed by Irving, Owen and Sterling [2]. In this new approach a cost function is chosen such that it represents the sum of the error module of the distances of the solution point to the measurement hyperplanes. The L.P. solution point lies actually on the point of intersection of n hyperplanes in n dimensional space. Irving et al had carried out a number of tests to compare the performance of the L.P. method with that of the WLS method for different networks. The tests were made for varying noise levels and degrees of redundancy ratio in the presence and absence of bad data. Redundancy ratio is defined here as the ratio of the number of the measurements to the number of state variables. The results of the tests indicated that the final solutions of the L.P. method, and the WLS were the same but the former was unchanged in the presence of some large measurement errors. Furthermore, the computer time requirements for the two methods were comparable for a redundancy ratio of about 1.5. It has also been shown in [3] that the WLAV technique and the L.P. estimation problem are equivalent. The difference is in their approach to reach the final solution. In the WLS approach the sequence of solution is estimation, bad data detection and identification. In the L.P. approach these steps are solved simultaneously.

In the original proposal the full Jacobian matrix for both nodal and branch mismatches was used and it seems that is is updated at the start of each Newton-Raphson iteration. Also no mention is made of any approximations for the calculation of the matrix elements. This is unattractive in terms of computation time and storage. These drawbacks could offset the balance of merits against the excellent bad data rejection property of the L.P. method. In the present paper, the decoupling technique is successfully applied to both nodal injections and line flows. Furthermore, the Jacobian submatrices are not only constant (being evaluated once only at the start of the first iteration) but are also approximated along the same lines as those of the fast decoupled technique. P-θ and Q-V are solved as two separate linear programming sub-problems. The programs are run alternately and the results from one program are used to modify the power mismatches of the other. This has resulted in a considerable reduction in computation time and storage. The accuracy of the final solution is unaffected.

STATE ESTIMATION PROBLEM

The general estimation consists of estimating the state vector [X] based on a set of observations [Z] in the presence of an error vector [w]. A mathematical model describing the functional relations between [Z], [X] and [w] is formulated.

This model is expressed in the form of a set of non-linear equations which relates the measurement [Z] and the true state vector $[\hat{X}]$.

$$[Z] = [\, f(\hat{X}) \,] + [w] \qquad (1)$$

The measurements vector [Z] can be nodal injections measurements or branch flow measurements. These are summarised in the following four equations.

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$$P_k = \sum_{m=1}^{n} V_k V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] \quad (2)$$

$$Q_k = \sum_{m=1}^{n} V_k V_m [G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)] \quad (3)$$

$$P_{km} = V_k V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] - V_k^2 [t_{km} G_{km} - G'_{km}] \quad (4)$$

$$Q_{km} = V_k V_m [G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)] + V_k^2 [t_{km} B_{km} - B'_{km}] \quad (5)$$

where

$G_{km} + jB_{km}$ = element of the nodal admittance matrix

t_{km} = $1/(1.0 + 0.01a)$

a = transformer tap position in per cent.

$G'_{km} + jB'_{km}$ = half line charging admittance between nodes k and m.

Solution algorithms for (2) and (4) are based mostly on a linearised approach. This can be grouped into the following two well-known linear matrix equations.

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\theta_m \\ \dfrac{\Delta V_m}{V_m} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \Delta P_{km} \\ \Delta Q_{km} \end{bmatrix} = \begin{bmatrix} R & U \\ T & S \end{bmatrix} \begin{bmatrix} \Delta\theta_m \\ \dfrac{\Delta V_m}{V_m} \end{bmatrix} \quad (7)$$

ΔP_k, ΔQ_k, ΔP_{km}, ΔQ_{km} are the differences between the measurement values and those that are obtained from (2) - (5). [H], [N], [J], [L], [R], [U], [S] and [T] are the submatrices of the full Jacobian maxtrix.

One of the most popular solution methods is that of the WLS and the method iterates on the following equation:

$$[\Delta X] = [[F]^T [C]^{-1} [F]]^{-1} [C]^T [\Delta Z] \quad (8)$$

where [C] is the diagonal covariance matrix. $[\Delta X]$ is $[\Delta\theta_m \ \ \frac{\Delta V_m}{V_m}]^T$ and $[\Delta Z]$ is composed of the mismatches of ΔP_k, ΔQ_k, ΔP_{km} and ΔQ_{km}. [F] is the jacobian matrix which is composed of the sub-matrices of (6) and (7).

The change in the state variables $[\Delta X]$ is calculated from (8) and is used to modify the measurement mismatches $[\Delta Z]$ which are in turn used to obtain another correction of the state variables. This process is repeated until convergence is reached. In the event that nodal voltages are measured, they can be used as a starting point in the iteration procedure. Estimation is only part of a complete estimator. A second important function is to detect and identify bad data. Detection of bad data is done through the J-index test [1]. The computed $J(\hat{X})$ is compared with a constant calculated from the chi-square distribution. If $J(\hat{X})$ exceeds this value, bad data is assumed to be present. Bad data can also be identified through the normalised residue vector or weighted residue vector [10]. In the next section a decoupled linear programming approach is described.

DECOUPLED LINEAR PROGRAMMING APPROACH

The linear programming approach to power system state estimation is a novel variation to the existing state estimation techniques. It has an excellent bad data rejection property. The original method has several drawbacks, the main one being the use of the full Jacobian matrix. Moreover, there is no indication in [2] to show that a constant matrix is used. It is therefore assumed that the Jacobian matrix is updated at the start of each Newton-Raphson iteration. Also, there is no mention of any approximations made in the calculation of the matrix elements in order to reduce computation time.

The first step in the decoupled formulation is to neglect the coupling submatrices [N], [J], [U] and [T] of (6) and (7) to yield

$$[\Delta P_k] = [H] \ [\Delta\theta_m] \quad (9)$$

$$[\Delta Q_k] = [L] \ \left[\frac{\Delta V_m}{V_m}\right] \quad (10)$$

$$[\Delta P_{km}] = [R] \ [\Delta\theta_m] \quad (11)$$

$$[\Delta Q_{km}] = [S] \ \left[\frac{\Delta V_m}{V_m}\right] \quad (12)$$

The nodal measurements (9) and (10) are then approximated along the same lines as those of the fast decoupled load flow [2] so that the final form of these equations is as follows:

$$\left[\frac{\Delta P_k}{V_m}\right] = [B_1] \ [\Delta\theta_m] \quad (13)$$

$$\left[\frac{\Delta Q_k}{V_m}\right] = [B_2] \ [\Delta V_m] \quad (14)$$

It is worth remembering that $[B_1]$ and $[B_2]$ are constant and highly sparse matrices.

The following assumptions and approximations are then applied to simplify the elements.

$$\cos(\theta_k - \theta_m) \doteq 1$$

$$B_{km} \gg G_{km} \sin(\theta_k - \theta_m)$$

$$V_k \doteq V_m \ ; \quad V_k^2 = V_k V_m$$

(11) and (12) are then simplified to yield the following:

$$[\Delta P_{km}] = [V_k V_m B_3] \quad [\Delta \theta_m] \tag{15}$$

$$[\Delta Q_{km}] = [V_k V_m B_4] \quad \left[\frac{\Delta V_m}{V_m}\right] \tag{16}$$

To avoid the need to update the Jacobian matrix at each iteration, the term $V_k V_m$ is transferred to the L.H.S. of these two equations. Furthermore, in order to make the R.H.S. of (16) compatible with that of (14), V_m is set to 1 p.u. The final equations for the line flows are:

$$\left[\frac{\Delta P_{km}}{V_k\ V_m}\right] = [B_3] \quad [\Delta\theta_m] \tag{17}$$

$$\left[\frac{\Delta Q_{km}}{V_k\ V_m}\right] = [B_4] \quad [\Delta V_m] \tag{18}$$

The matrices $[B_3]$ and $[B_4]$ are also constant, highly sparse and consist only of two non-zero elements per row.

The estimation problem can now be decoupled into LP sub-problems. The voltage angles can be estimated from the combination of (15) and (17) to give the P-θ sub-problem, while the voltage magnitudes can be estimated from the combination of (16) and (18) to form the Q-V sub-problems. These two sub-problems can be represented by the following equations,

$$\begin{bmatrix} \Delta P_k \\ \\ \dfrac{\Delta P_{km}}{V_k V_m} \end{bmatrix} = \begin{bmatrix} B_1 \\ \\ B_3 \end{bmatrix} \begin{bmatrix} \Delta\theta_m \end{bmatrix} \; : \; \begin{bmatrix} \Delta Q_k \\ \\ \dfrac{\Delta Q_{km}}{V_k V_m} \end{bmatrix} = \begin{bmatrix} B_2 \\ \\ B_4 \end{bmatrix} \begin{bmatrix} \Delta V_m \end{bmatrix} \qquad (19) \text{ and } (20)$$

It can be seen that the original formulation in [2] is a LP problem based on a tableau formed from (6) and (7). The tableau contains the sub-matrices [H], [N], [J], [L], [R], [U], [T] and [S]. The tableau is large and the elements of these sub-matrices are not constant which means they need to be updated at each iteration. In the authors proposed modification, two tableaus are needed, one for each sub-problem, and they are formed from (19) and (20) respectively. By comparing the equations it can be seen that these tableaus are much smaller than the original formulation, even when they are combined together. Furthermore the elements in these two tableaus are constant while the sparsity of the matrices remains the same. This resulted in a reduction in computer storage and time. Either (19) or (20) can be rewritten as the following final form:

$$[\Delta Z] = [F] \quad [\Delta X] \tag{21}$$

The final form of the algorithm is now developed on (21) which can be used for either the P-θ L.P. sub-problem or the Q-V L.P. sub-problem. In the presence of noise and meter errors, $[\Delta Z]$ in (21) deviates from the ideal mismatch of the measurement vector. This deviation could be positive or negative. To cater for this change, slack variables are added to and subtracted from the R.H.S. of (21) to yield:

$$[\Delta Z] = [F] \quad [\Delta X] + [A] - [D] \tag{22}$$

where [A] and [D] are vectors of slack variables representing the errors in the measurements and are always positive or zero. The change in [ΔX] of the state vector is not always positive as well. To satisfy the requirements of the LP algorithm, a constant E_k is added to each ΔX_k to give $\Delta X_k'$. This constant is so chosen such that it is sufficiently large to ensure that the resultant $\Delta X_k'$ is always positive. Another approach is to use separate variables which means that one variable is split into two variables. This approach is obviously unattractive as it effectively doubles the number of variables. (22) is then modified to become

$$[\Delta Z'] = [F] \quad [\Delta X'] + [A] - [D] \tag{23}$$

where $\quad [\Delta Z'] = [\Delta Z] + [F] \quad [E]$

It is worth noting that $[\Delta Z]$ are the mismatches that are defined in (19) and (20).

To arrive at the best or optimal estimate of the state variables, the errors or the deviations from the true values (represented here by the slack variables A_k and D_k) are minimised. The cost function is therefore chosen as:

$$\min \sum_{k=1}^{k=m} w_k \ (A_k + D_k) \tag{24}$$

where w_k is a weighting factor for the k^{th} measurement such that measurements which are known to be more reliable are given more weight. These weighting factors can be calculated from the known accuracy of the instrumentation and telemetry system. The approach is exactly the same as the one used for calculating the weighting factors for the WLS. Full details can be found in [1].

The sub-problem is then defined as the minimization of (24) subject to the set of linear constraints of (23) which consists of m constraints and (n + 2m) variables, where n is the number of state variables and m the number of measurements.

The size of the sub-problem may be reduced by rewriting the equality constraints of (23) as inequalities. Since A_k is by definition either zero or positive, the k^{th} equality constraint of (23) may be rewritten as an inequality,

$$F_{k1}\,\Delta X_1' + F_{k2}\,\Delta X_2' \ \ldots \ F_{kn}\ \Delta X_n' - D_k \leq \Delta Z_k' \tag{25}$$

Furthermore, the sum of the slack variables $A_k + D_k$ may be written as

$$A_k + D_k = 2D_k + \Delta Z_k' - F_{k1}\,\Delta X_1' - F_{k2}\,\Delta X_2' \ \ldots - F_{kn}\ \Delta X_n' \tag{26}$$

The minimization sub-problem can now be redefined as follows

$$\min \sum_{k=1}^{k=m} w_k (2D_k + \Delta Z_k' - F_{k1}\,\Delta X_1' - F_{k2}\,\Delta X_2' \ \ldots F_{kn}\,\Delta X_n') \tag{27}$$

subject to the inequality constraint set of

$$[F] \quad [\Delta X'] - [D] \leqslant [\Delta Z'] \tag{28}$$

and the non-negativity constraints of

$$[\Delta X'] \geqslant 0$$

and

$$[D] \geqslant 0$$

All the voltage angles listed in Figures 1 - 5 are given in radians.

Bus code p-q	Impedance Z_{p-q}	Line charging Y'_{p-q}	Line flow P_{p-q} MW	MVAR	Bus Code P	Busbar voltage (per unit)		Net busbar power (generation positive)	
						Magnitude	Angle	MW	MVAR
1-2	0.02+j0.06	0.0+j0.06	88.8	-8.6	1	1.060	0.000	(129.5)	(-7.5)
1-3	0.08+j0.24	0.0+j0.05	40.7	1.1	2	1.047	-0.049	20.0	20.0
2-3	0.06+j0.18	0.0+j0.04	24.7	3.5	3	1.024	-0.087	-45.0	-15.0
2-4	0.06+j0.18	0.0+j0.04	27.9	3.0	4	1.024	-0.093	-40.0	- 5.0
2-5	0.04+j0.12	0.0+j0.03	54.8	7.4	5	1.018	-0.107	-60.0	-10.0
4-5	0.08+j0.24	0.0+j0.05	6.3	-2.3					

Figure 2: DATA FOR 5-BUSBAR NETWORK

Bus code p-q	Impedance Z_{p-q}	Line charging Y'_{p-q}	Line flow P_{p-q} MW	MVAR	Bus Code P	Busbar voltage (per unit)		Net busbar power (generation positive)	
						Magnitude	Angle	MW	MVAR
1-2	0.0015+j0.0118	0.0+j0.3770	320.67	-91.19	1	1.0006	0.0000	(526.09)	(-164.69)
1-6	0.0024+j0.0208	0.0+j0.1225	169.51	-22.42	2	1.0051	-0.0387	-94.80	36.00
1-19	0.0032+j0.0277	0.0+j0.2968	35.91	-50.98	3	0.9931	-0.0949	-166.51	-27.14
2-3	0.0007+j0.0338	0.0+j0.0000	166.71	36.89	4	0.9895	-0.1008	-73.86	-8.38
2-6	0.0014+j0.0123	0.0+j0.0743	-20.85	35.59	5	0.9895	-0.1008	-73.86	-8.38
2-8	0.0029+j0.0152	0.0+j0.4405	334.26	-119.85	6	1.0006	-0.0356	0.00	0.00
2-14	0.0030+j0.0106	0.0+j0.2733	-255.87	17.36	7	0.9783	-0.1900	-165.47	-44.21
4-6	0.0020+j0.0875	0.0+j0.0000	-73.86	-8.38	8	1.0116	-0.0915	-165.00	155.63
5-6	0.0020+j0.0875	0.0+j0.0000	-73.86	-8.38	9	1.0201	0.1112	-68.99	1.66
7-8	0.0010+j0.0591	0.0+j0.0000	-165.47	-44.21	10	1.9758	-0.0701	-77.48	-15.79
9-22	0.0030+j0.0988	0.0+j0.0000	-68.99	1.66	11	0.9846	-0.0687	-59.67	-13.15
10-15	0.0030+j0.1671	0.0+j0.0000	-77.48	-15.79	12	1.0156	0.0640	0.00	0.00
11-16	0.0030+j0.1671	0.0+j0.0000	-59.67	-13.15	13	1.0194	0.0444	0.00	0.00
12-13	0.0028+j0.0193	0.0+j0.0748	100.21	-37.40	14	1.0098	-0.0111	0.00	0.00
12-15	0.0007+j0.0046	0.0+j0.0177	77.72	25.24	15	1.0139	0.0607	0.00	0.00
12-22	0.0094+j0.0643	0.0+j0.2491	-177.93	12.16	16	1.0138	0.0311	0.00	0.00
13-14	0.0123+j0.0452	0.0+j0.2787	123.83	-23.08	17	1.9983	-0.0485	-62.26	-15.80
13-16	0.0035+j0.0238	0.0+j0.0924	59.91	10.92	18	1.0097	-0.0114	0.00	0.00
13-20	0.0030+j0.1225	0.0+j0.0000	24.96	-3.29	19	1.0095	-0.0110	0.00	0.00
13-21	0.0030+j0.1225	0.0+j0.0000	24.97	-3.27	20	1.0231	0.0149	-24.94	4.04
13-23	0.0040+j0.0206	0.0+j0.0971	-133.77	-13.02	21	1.0230	0.0149	-24.95	4.02
14-18	0.0001+j0.0010	0.0+j0.0058	31.01	8.65	22	1.0227	0.1766	250.00	-15.05
14-19	0.0001+j0.0010	0.0+j0.0057	-4.51	30.23	23	1.0266	0.0704	300.00	-7.95
14-23	0.0123+j0.0510	0.0+j0.2844	-162.34	-1.85					
17-18	0.0030+j0.1217	0.0+j0.0000	-30.98	-7.98					
17-19	0.0030+j0.1217	0.0+j0.0000	-31.28	-7.82					

Figure 2: DATA FOR 23-BUSBAR NETWORK

Bus code P	Exact busbar voltages (load flow values)		Measurements ± 5%	
	Magnitude	Angle	Magnitude	Angle
1	1.060	-0.000	1.060	0.000
2	1.047	-0.049	1.046	-0.051
3	1.024	-0.087	1.022	-0.089
4	1.024	-0.093	1.021	-0.096
5	1.018	-0.107	1.014	-0.111

Figure 3: 5-BUSBAR SYSTEM ESTIMATES

Bus code p	Exact busbar voltages (load flow values)		10 Bad data points		20 Bad data points		30 bad data points	
	Magnitude	Angle	Magnitude	Angle	Magnitude	Angle	Magnitude	Angle
1	1.0006	0.0000	1.0006	0.0000	1.0006	0.0000	1.0006	0.0000
2	1.0051	-0.0387	1.0051	-0.0387	1.0051	-0.0387	1.0124	-0.0220
3	0.9931	-0.0949	0.9931	-0.0949	1.0023	-0.0946	1.0097	-0.0769
4	0.9895	-0.1008	0.9895	-0.1008	0.9895	-0.1007	1.0079	-0.0188
5	0.9895	-0.1008	0.9895	-0.1008	0.9895	-0.1008	1.0050	-0.0748
6	1.0006	-0.0356	1.0006	-0.0356	1.0006	-0.0356	1.0079	-0.0188
7	0.9783	-0.1900	0.9783	-0.1900	0.9739	-0.1903	0.9913	-0.1484
8	1.0116	-0.0915	1.0116	-0.0915	1.0074	-0.0909	1.0240	-0.0523
9	1.0201	0.1112	1.0201	0.1112	1.0201	0.1112	1.0223	0.1125
10	0.9758	-0.0701	0.9758	-0.0701	0.9758	-0.0701	0.9785	-0.0674
11	0.9846	-0.0687	0.9846	-0.0686	0.9846	-0.0686	0.9870	-0.0664
12	1.0156	0.0640	1.0156	0.0640	1.0156	0.0640	1.0179	0.0655
13	1.0194	0.0444	1.0194	0.0444	1.0194	0.0444	1.0217	0.0460
14	1.0098	-0.0111	1.0098	-0.0111	1.0098	-0.0110	1.0171	0.0054
15	1.0139	0.0607	1.0139	0.0607	1.0139	0.0607	1.0162	0.0622
16	1.0138	0.0311	1.0138	0.0311	1.0138	0.0311	1.0161	0.0328
17	0.9983	-0.0485	0.9983	-0.0485	0.9983	-0.0484	1.0076	0.0428
18	1.0097	-0.0114	1.0097	-0.0114	1.0097	-0.0113	1.0170	0.0057
19	1.0095	-0.0110	1.0095	-0.0110	1.0095	-0.0110	1.0168	0.0055
20	1.0231	0.0149	1.0231	0.0149	1.0231	0.0150	1.0209	0.0167
21	1.0230	0.0149	1.0230	0.0149	1.0230	0.0149	1.0253	0.0167
22	1.0227	0.1766	1.0227	0.1766	1.0227	0.1766	1.0249	0.1777
23	1.0266	0.0704	1.0266	0.0704	1.0266	0.0704	1.0289	0.0719

Figure 4 : 23 BUSBAR NETWORK ESTIMATES (Bad data rejection)

Bus code	Measurement		10 Bad data		20 Bad data		30 Bad data	
p	MW	MVAR	MW	MVAR	MW	MVAR	MW	MVAR
3		-27.14		0.00		0.00		0.00
4	-73.86		0.00		0.00		0.00	
5		-8.38				0.00		0.00
7		-44.21						0.00
8	-165.00						0.00	
9	-68.99		0.00		0.00		0.00	
10		-15.79						15.79
11		-13.15		13.15		13.15		13.15
15	0.00						50.00	
16	0.00				50.00		50.00	
17	-62.26				62.26		62.26	
18		0.00				50.00		50.00
19		0.00						50.00
20	-24.94				24.94		24.94	
21		4.02		50.00		50.00		50.00
2-3	166.71						0.00	
2-8		-119.85				0.00		0.00
4-6		-8.38				0.00		0.00
5-6	-73.86						73.86	
7-8	-165.47		0.00		0.00		0.00	
9-22		1.66						50.00
10-15	-77.48				0.00		0.00	
11-16	-59.67						59.67	
12-15		25.24		-25.24		-25.24		-25.24
13-14	123.83		0.00		0.00		0.00	
13-20		-3.29						50.00
13-21	24.97				-24.97		-24.97	
14-18	31.01		-31.01		-31.01		-31.01	
14-23		-1.85				50.00		50.00
17-18		-7.98		7.98		7.98		7.98

Figure 5: BAD DATA POINTS IN 23 BUSBAR NETWORK

This is a linear programming problem of m constraints and (n + m) variables.

The state estimation problem has now been decoupled into two separate L.P. sub-problems. These two are solved in a sequential manner such that the results obtained from one are used to update $[\Delta Z']$ of the other.

COMPUTATIONAL RESULTS AND DISCUSSION

Tests were carried out on two networks - a 5 bus, and a 23 bus system, the latter being part of the EHV network of the North of Scotland Hydro Electric Board. The linear programming subroutine used is a general standard package. Initial tests were performed with no bad data and with a flat voltage start. The solutions between that of a load flow, the WLS and the decoupled LP approach agreed to four significant figures. Extensive further testings for different loading conditions from different starting points showed consistent convergence and the same accuracy. This shows that decoupling the problem does not affect the final solution. The criterion for convergence is chosen as the change in the state vector itself. Convergence is assumed to be reached if the change in the state vector between two successive iterations is less than a preset tolerance. The decoupled LP formulation does not take into account $|V|$ measurements. In the event that $|V|$ measurements are available, they can be used as starting points.

To ensure speedy convergence and the reliability of the L.P. sub-problem, a small scalar was added to the elements of the main diagonal of the Jacobian matrix for the nodal power flow measurements. Initially, these elements were increased by 5%. This was found to give reliable convergence. Extensive tests were then performed. The best results were obtained when all the elements were increased by a fixed scalar quantity, typically either 1 or 2. This scalar does not affect the final solution point.

During successive half Newton-Raphson iterations, the rate of convergence of the P-θ sub-problem is sometimes found to be faster than the corresponding Q-V sub-problem. Three schemes were then tested: (1θ, 1V), (1θ, 2V) and (1V, 1θ). From experience consistent fast overall convergence is obtained with the schemes (1θ, 1V) and (1θ, 2V). Further tests indicate that no benefit can be gained by iterating more than once on the Q-V sub-problem. Hence the scheme of (1θ, 1V) is recommended.

Percentage Errors

To simulate meter inaccuracies, a percentage error was added to all the measurements. Meter inaccuracy, other than meter malfunction is assumed to lie within $\pm$ 5%. Statistically, it is normal to expect that some meters have positive percentage errors and others to have negative percentage errors. In order to achieve this, random errors of $\pm$ 5% are injected into each measurement in the 5-node test system. One set of the test results is shown in Figure 3, and the following deviations from the true values are retained.

- maximum error in voltage magnitude $<$ 0.4%
- maximum error in angle values = 0.17 degree.

Effect of Bad Data

There is no guarantee in the LP approach that the 'basis' will not include bad data measurements. Theoretically it is possible to have bad data points equal to the number of redundant measurements without affecting the final solution. In practice, this is not usually the case and its effects are discussed in this section.

Any network could be looked upon as a number of subnetworks interconnected by a number of lines. A special case is that of a node or number of nodes connected to the rest of the network by a radial feeder. For such subnetworks there is a tolerable limit to the number of lost measurements or bad data points, starting from the interconnecting lines and going towards the subnetwork. This limit occurs when the number of good measurements within the subnetwork is equal to the number of its state variables. This is the maximum number of bad data permissible. Still, all the bad data will be rejected and the true solution point will be reached. Beyond this limit the redundancy ratio within the subnetwork fails below unity and some of the bad data will affect the final solution. In this latter case, an isolated island or network will be formed with incomplete information for the program to reach the correct solution. Thus the number of tolerable bad data points depends on how well interconnected the network is and how evenly the bad data is distributed.

Experience shows that introducing one or two bad data points to the 5 bus system does not affect the final solution. The bad data points are indicated in the program by their associated slack variables which assume the values of the power mismatches. The error power mismatches also remain unchanged and do not reduce to zero throughout the iterations. Figures 4 and 5 show the effects of randomly introducing 10, 20 and 30 bad data points to the 23 bus system. In the presence of bad data, the time per iteration is reduced but convergence is slower.

EFFECTS OF TOPOLOGICAL ERROR AND REDUNDANCY RATIO

A case which is of prime importance is that of the tripping out of a transmission line without being detected when a state estimation is to be performed. Normally, the tripping should be noticed immediately by the control engineers but, in the event that it is not, two cases are being considered here. The first case is the tripping out of a lightly loaded line and the second case is that of a heavily loaded line. A load flow solution is obtained for each case with the line concerned disconnected. Then a state estimation for the system is performed for each of the two cases and in each case the tripped line is assumed to be in service (incorrect topology of the network) with the measurement from the line set to zero. The maximum errors in voltage magnitudes and angles are given in Figure 6.

Maximum error / Line disconnected	Busbar voltage magnitude (per cent)	Busbar voltage angle (degree)
17 - 19 (light loaded)	1.31	2.23
13 - 23 (heavily loaded)	1.4	3.93

Figure 6: BUSBAR VOLTAGE MAGNITUDE AND ANGLE ERROR

Theoretically, the speed of convergence is increased with the decrease of redundancy ratio. It has been pointed out in [4] that for economic reasons high redundancy ratio is not reaslistic in practice. At slightly greater than unity redundancy ratio, the estimator can still give an accurate estimate provided that the system is not ill-conditioned and that the metering strategy is correct. With the decoupled L.P. approach no estimation is possible when the redundancy ratio falls below unity. In order to give an insight into the relationship between redundancy ratio and computation time, several test runs had been performed. Figure 7 gives the results of the tests. It appears that the relationship is almost linear. The actual computation time is not important as it has been pointed out that the L.P. subroutine used in the test is a general purpose one. Undoubtedly the overall computing time will be reduced if a special LP package is written specifically for state estimation application.

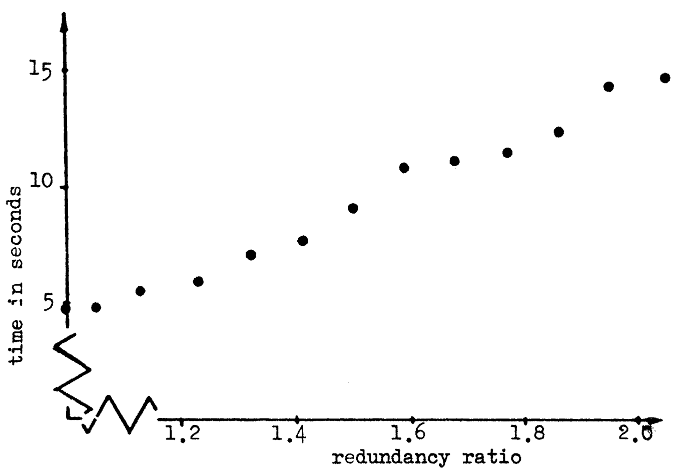

Figure 7: Time/redundancy ratio

CONCLUSIONS

The decoupling of P-θ, Q-V has been effectively used to solve the L.P. state estimation problem with substantial reduction in storage and computing time. This decoupling approach does not affect the final solution point of the state variable.

The main attractions of the present approach may be summarised as follows:

(1) The effect of decoupling leads to a reduction in computer storage and computing time.

(2) A fixed Jacobian matrix is used and is calculated only once at the start of the first iteration.

(3) Approximated Jacobian elements without sine or cosine terms greatly reduce the time for the evaluation of the Jacobian matrix.

(4) Bad data points are automatically rejected and indicated by their associated slack variables, and these in general do not grossly affect the final solution point provided they are less than (m-n).

(5) A large redundancy ratio is not necessary for an accurate estimate. Thus, with a good metering strategy, the percentage of measurement redundancy over unity should be slightly greater than the percentage of bad data usually encountered in practice, and this is generally low in practice.

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REFERENCES

(1) Lo, K.L., Ong P.S., McColl, R.D., Moffatt A.M., Sulley J.L. "Development of a static state Estimator Part I. Estimation and Bad Data Suppression", para 85 WM 056-9, Winter Meeting, February 1983.

(2) Irving, M.R., Owen, R.C. and Sterling, M.J.H., "Power system state estimation using linear programming", Proc.IEE, Vol.125, No.9, Sept. 1978, pp. 879-885.

(3) Kotinga W.W., Vidyasagan M., "Bad Data rejection properties of weighted least absolute value techniques applied to static estimation", IEEE Trans. PAS. 101, April 1982, pp. 844 - 851.

(4) Lo, K.L., On, P.S., McColl, R.D., Moffatt, A.M., Sulley, J., Moyes, I., "Power System State Estimation: a method for metering reinforcement", Paper 82 WM 138-6, Winter Meeting, February 1982.

(5) Stott, B. and Alsac, O., "Fast decoupled load flow" IEEE Trans., PAS, Vol.93, 1974, pp.859-869.

(6) Schweppe, F.C. and Wildes, J., "Power system static-state estimation, Part I: exact model", IEEE Trans., PAS, Vol.89, No. 1, Jan. 1970, pp. 120-125.

(7) Schweppe, F.C.and Rom, D.B., "Power system static-state estimator, Part II: approximate model", pp. 125-130.

(8) Schweppe,F.C., "Power system state estimation, Part III: Implementation" pp. 130-135.

(9) Smith, O.J.M., "Power system state estimation"' IEEE Trans., PAS, Vol.89, No. 3, March 1970, pp. 363-379.

(10) Merrit, H.M. and Schweppe, F.C., "Bad data suppression in power system static state estimation", IEEE Trans. PAS, Vol.90, 1971, pp. 2718-2725.

(11) Vemura, K., "Approximated Jacobians in Newton's power flow method", Proc. Power System Computation Conf., Paper 1.3/2, Sept. 1972.

(12) Tripathy, S.C., Rao, N.D. and Kumar, A., "Real time monitoring of power systems susing fast decoupled load flow", Proc. IEE, Vol. 124, No. 7, July, 1977, pp. 602-606.